ARITHMETIC REASONING

For most people, the math sections of Examinations are the most difficult. This section is one of the least popular, as it consists solely of mathematical word problems. Yet we've found that people can drastically improve their scores by practicing with word problems before the exam, and consciously cultivating good problem solving habits.

There are usually fifteen arithmetic reasoning questions. Most are of moderate difficulty, a few are pretty easy and two or three are sometimes very tricky. If you get stuck on one of the more difficult questions, you can put a check mark next to it and come back to it later if there's time. On most exams people don't usually run out of time, just patience. This is understandable, but points given for correctly answering one or two of these difficult questions may sometimes make a difference in ones place on a promotion list, so it's good to go back and try different approaches to solving a problem, rather than just guessing.

We suggest you check the answer key after every three questions. (Be sure to spend enough time on each question. Don't just give up quickly and turn to the answer key; (you won't get as much out of the process if you do). If you've missed any questions, consult the Self Study Guide and go through the explanation thoroughly before you continue on to the next question. (Don't worry if you don't do well at first, it's been a long time since most people have answered these types of questions). If your exam won't allow you to use a calculator, it's a good idea to practice these questions without a calculator to increase speed, accuracy and confidence. Remember, these questions are not easy for most people. They may seem really difficult at first and some may seem impossible. If, however, a real effort is made to practice and learn from mistakes, scores in this area can improve considerably. We suggest you do these problems again a week before the exam.

Good luck!

ARITHMETIC REASONING

1. If Jean's weekly income doubled she would be making $120 a week more than Barbara. Jean's weekly income is $80 more than half of Betty's. Betty makes $200 a week.

How much does Barbara make?

a. $180 c. $240
b. $200 d. $360

2. A conference with 3600 participants gathers in Albany. One of every twelve people attending the conference who have ordered meals has special dietary needs. Half of those attending the conference signed up for meals. How many have special dietary needs?

a. 266 c. 150
b. 133 d. 300

3. It costs $360 for Office X's service contract with a typewriter company to service 18 typewriters for six months. At this same rate, how much would it cost Office X to service six typewriters for three months?

a. $80 c. $90
b. $75 d. $60

4. In December, an office spent $480, or 15% of its non-personnel expenses that month, for postage. What were its total non-personnel expenses for December?

a. $3200 c. $5520
b. $552 d. $7200

5. Catherine bought an equal number of $11.00, $9.00, and $8.00 tickets for a concert. She spent $196 for all of the tickets. How many of each did she buy?

a. 6 c. 8
b. 7 d. Cannot be determined from information given

6. Agency Y employs 13,800 people. Of these, 42% are male, and 50% of

the males are age 30 or younger. How many males are there in Agency Y who are older than 30?

a. 5796 c. 3471
b. 2898 d. 2910

7. A machine can collate 126 books, each with 400 page, in 14 days. If it continues to collate at this same rate, how many 400 page books could it collate in 30 days?

a. 256 c. 248
b. 290 d. 270

8. A typewriter and a dictation machine cost a total of $840. If the typewriter cost $360 more than the dictation machine, how much did the dictation machine cost?

a. $480 c. $240
b. $440 d. $280

9. A cabinet maker has a round piece of wood 1/2" in diameter and 3/4 yards long. She needs half the length for the back of a chair and the remaining piece for 3/4" pegs. How many pegs will she have?

a. 18 c. 10
b. 9 d. 8/9

10. Robin can wallpaper a room in four hours. Susan can wallpaper the same room in seven hours. How long will it take them to wallpaper the room if they work together?

a. 4.5 hours c. 5.5 hours
b. 3.5 hours d. 2.5 hours

11. Mary and Alice jog 3 miles each evening. If they run at a constant rate and it takes Mary 40 minutes while Alice finishes in half an hour, how

much distance does Mary have left when Alice finishes?

a. 1 mile c. 2/3 mile
b. 3/4 mile d. 1.33 miles

12. As a fund raiser , a community organization buys tickets to the theater to resell 25% above cost. They buy 50 eight dollar tickets, 25 ten dollar tickets and 25 fifteen dollar tickets. If they sell all but three of the ten dollar tickets, how much money have they made?

a. $218.75 c. $248.75
b. $1273 d. $1243.75

13. If a couch cost $640 after a 20% discount, what was its original price?

a. $769 c. $800
b. $512 d. $780

14. A salesperson traveled 145 miles Monday, 72 miles Tuesday, and 98 miles Wednesday for $2300 worth of sales. If the business pays 21¢ a mile for gas and vehicle maintenance, approximately what percent of sales for the three days went to gas and vehicle maintenance?

a. 3% c. 6%
b. 8% d. 1%

15. If one of every eight junior year students at a high school takes Latin, approximately what percent of a junior year class of 650 took Latin?

a. 6 c. 81
b. 14 d. 13

16. The proposed budget for a new social service program is $102,000. The budget states that the project has secured $14,500 worth of transportation services and $1,200 worth of office equipment as in kind contributions. What percent of the budget has been secured in kind?

a. 21.4 c. 20.9
b. 15.4 d. 13.1

17. The purpose of the program in Question 16 is to distribute 250,000 pounds of food to disadvantaged persons. Approximately how many pounds of food will be distributed for each dollar budgeted for the program?

a. 3.73 c. 2.45
b. 2.15 d. 2.95

18. A community cannery charges 15¢ per quart and 25¢ for 2 pints for processing. People using the cannery can purchase jars there at 15% discount off the regular price of $4.25 per case of a dozen quart or pint jars. How much will it cost to can 76 quarts of tomatoes and 20 pints of jelly if the jars are bought at the cannery? (Jars are not sold individually).

a. $46.39 c. $32.49
b. $37.14 d. $13.90

19. A pharmacist combines ingredients x, y and z in a ratio of 1:2:7 to produce cough medicine. How many ounces of the second ingredient, ingredient y, is needed to make a 12 ounce bottle of the medicine?

a. 2.4 c. 1.2
b. 8.4 d. 3.6

20. Agency Y served 187,565 people in 1981. If the agency served 210,515 people in 1982, this reflected an increase of:

a. 19.10% c. 12.2%
b. 15.6% d. 10.9%

21. The number of people attending a weekly training program in the month of January averaged 116 people. If there were 105 people attending the first week, 106 the second, and 125 the third, how many people attended the fourth week?

a. 118 c. 130
b. 128 d. 124

22. It takes 16 typists 11 days to complete a project. How long would it take 10 typists, if they worked at the same rate to complete the same project?

a. 17.6 days c. 6.9 days
b. 6.8 days d. 18.4 days

23. If the sum of two numbers is 280, and their ratio is 7:3. then the smaller number is

a. 28 c. 56
b. 84 d. 196

24. The population of Metropolis county in 1982 is 130% of its population in 1972. The population in 1972 was 145,000. What was the population in 1982?

a. 196,425 c. 111,539
b. 174,612 d. 188,500

25. A car travels 50 miles an hour, and a plane travels 10 miles a minute. How far will the car travel when the plane travels 500 miles?

a. 50.4 miles c. 41.6 miles
b. 37.5 miles d. 39.7 miles

26. In a university with 2000 students the student-faculty ratio is 16:1. If

18% of the faculty have completed some of their own study at the university, approximately how many have not?

a. 119 c. 23
b. 127 d. 103

27. A discount house advertises that they sell all merchandise at cost plus 10%. If Jane buys a TV set for $300, approximately what is the stores profit?

a. $30.00 C. $27.27
b. $27.00 d. $32.26

28. From 6 p.m. until midnight, the temperature dropped at a constant rate. From midnight until 1 a.m., it dropped 8°. If at 6 p.m., the temperature was 54° and by 1 a.m., it was 37°, what was the temperature at 10 p.m.?

a. 46° c. 45°
b. 48° d. 49°

29. One eighth of a half gallon carton of ice cream has been eaten. The remainder is divided among three people. Approximately what percentage of a gallon does each person get?

a. 14.6% c. 29.2%
b. 11.3% d. 18.1%

30. On a promotional exam a woman scored 143 on a scale of 0-160. Her score converted to a scale of 0-100 is approximately:

a. 89 c. 91
b. 70 d. 84

31. A woman paid a tax of $88.00 on property assessed at $28,000. Her

neighbor, assessed at the same rate. paid a tax of $110. What was the assessed value of the neighbor's house?

a. $22,400 c. $35,000
b. $32,400 d. $31,000

32. If Janet can build 22 tables in 14 days, and Anne can build 22 tables in 16 days, approximately how long will it take them to build 22 tables together?

a. 9.5 days c. 15 days
b. 7.5 days d. 8 days

33. Cynthia loaned $35 to Mary. But Cynthia borrowed $14 from Jean, and $16 from Emily Emily owes $17 to Jean and $9 to Mary. One day they got together to settle their accounts. Who left with $10 less than she came with?

a. Cynthia c. Mary
b. Jean d. Emily

34. How many square tiles, each 12 inches on a side, will Ozzie need to cover a floor that is 11 feet wide and 18 feet long?

a. 99 c. 163
b. 150 d. 198

35. A car has depreciated to 72% of its original cost. If the car is presently valued at $3245, approximately what was its original cost?

a. $5219 c. $4507
b. $5582 d. $2336

36. The sales tax on a typewriter is $13.41 and the sales tax rate is 4%. The purchase price, before the tax was added, was:

a. $335.25 c. $279.10
b. $536.40 d. $317.50

37. What is the interest on $600 at 8% for 30 days?

a. $4.00 c. $7.50
b. $11.52 d. $4.80

38. A garden is 30 feet by 40 feet. A fence is built around the garden at a cost of $1.75 per foot of fencing. What was the cost of the fencing?

a. $133.33 c. $210.00
b. $245.00 d. $122.50

39. A tax analyst earns four times as much in April as in each of the other months. What part of her entire year's earnings does she earn in April?

a. 4/11 c. 4/15
b. 1/3 d. 4/13

40. A train travels 70 miles when a bus travels 50 miles. How many miles will the train travel when the bus travels 60 miles?

a. 40 c. 90
b. 78 d. 84

METRIC ARITHMETIC REASONING

1. How many square tiles, each 300 mm on a side, will Jane need to cover a hall floor that is 120 cm wide and 3.6 m long?

a. 480 b. 520

c. 490 d. 48

2. A property is 100 m by 0.337 km. A fence is built around this property at a cost of $1.75 per foot of fencing. What wm the cast of the fencing?

a. $5,018 b. $764.75
c. $502 d. $2,009

3. A car travels 80 km/hr and a plane travels 16 000 m/min. How far will the car travel when the plane travels 800 km?

a. 80.6 km b. 66.7 km
c. 60.0 km d. 63.5 km

4. A pharmacist combines ingredients x, y, and z in a ratio of 2:3:5 to produce cough medicine How many milliliters of the second ingredient, ingredient y, is needed to make a one liter bottle of the medicine?

a. 0.3 b. 150
c. 300 d. 500

5. A cabinet maker has a dowel of wood 13 mm in diameter and 0.800 m long. She needs half the length for the back of the chair and the remaining piece for 37 mm pegs. Pegs greater than 13 mm in diameter or shorter than 37 mm will not work. How many good pegs can she make?

a. 108 b. 11
c. 10 d. 109

ANSWER KEY

ARITHMETIC REASONING

1.	c	21.	b
2.	c	22.	a
3.	d	23.	b
4.	a	24.	d
5.	b	25.	c
6.	b	26.	d
7.	d	27.	c
8.	c	28.	b
9.	a	29.	a
10.	d	30.	a
11.	b	31.	c
12.	a	32.	b
13.	c	33.	d
14.	a	34.	d
15.	d	35.	c
16.	b	36.	a
17.	c	37.	a
18.	a	38.	b
19.	a	39.	c
20.	c	40.	d

METRIC ARITHMETIC REASONING

1.	d
2.	a
3.	b
4.	c
5.	c

SELF STUDY GUIDE

ARITHMETIC REASONING

You should consult this guide whenever you miss a question or aren't sure why you got the answer you did.

You shouldn't get discouraged if you seem totally lost at first. With practice you will improve. In many cases these questions require using methods you may not have used in years, if ever. We have tested this, guide with many people, however, and all of them have been able to improve their ability to answer word problems by conscientiously using it. We don't mean to suggest that sometimes it won't be hard work - you may need to re-work and re-read some of the problems many times before they make sense. You will get out of this guide the fruits of whatever effort you put in, and perseverance in problem solving is always critical.

No knowledge of advanced math is required, and we have kept our explanations free of jargon and intimidating formulas. Basically, what you need is a knowledge of basic math and perseverance. In explaining the answers, we briefly review working with fractions, percentages and ratios.

It's also important to remember that there are often many ways to do a particular problem. We are presenting methods that are the easiest for most people. If you have a different approach, and you consistently get the right answer using it, there's certainly no need to change.

Good luck!

ARITHMETIC REASONING

1. The answer is C. This kind of question is difficult unless you break it down into parts, and solve it step by step. It's only in the next to last sentence that we're actually given someone's salary. We're told that Betty makes $200 a week. The question asks for Barbara's salary, but it becomes clear after careful reading that we can't find Barbara's salary unless we find Jean's. Jean's weekly income is $80 more than half of Betty's.

Jean's salary is $80 more than half of $200, so Jean's salary is $80 more than $100, or $180. The first sentence tells us that if Jean's weekly income doubled she would be making $120 a week more than Barbara. Two times Jean's salary of $180 would be $120 a week more than Barbara's salary. So, two times $180, which equals $360, would be $120 a week more than Barbara's salary. $360 is $120 more than what number? $360 - $120 - $240, Choice C.

2. Choice (C) is the answer: It's important in a question like this to identify and break down the information you are given. From the question, we know:

    1. 3600 people are attending the conference

    2. 1/2 of the 3600 people have ordered meals

    1. One out of 12 (or 1/12) of those who ordered meals has special dietary needs

It's important to remember that in order to solve this we must keep in mind that only half of the attendees have ordered meals. So half of the 3600 people, or 1800, have ordered meals. Of these, one out of every twelve has special dietary needs, so 1/12 of the 1800 people who signed up for meals have special dietary needs. There are many ways to solve this problem from this point on. "One way is to simply multiply 1800 by 1/12 to find the answer. (To multiply fractions, multiply the numerators by each other, and the denominators by each other.)

$$1/12 \times 1800 = 1/12 \times 1800/1 = 1800/12 = 150$$

So, 150 of those who have ordered meals have special dietary needs, Choice (C). Or you could have used decimals. 1/12 is expressed in decimal form as .0833 (to find the decimal form of a fraction, divide the numerator, the top number, by the denominator, the bottom number). 1800 x 1/12 = 1800 x .0833 = 149.94 or 150 people. (It comes out a little unevenly because the decimal has been rounded off.) You also could have set up a ratio, comparing those with special needs who ordered meals to all who ordered meals.

A Review of Ratios

Ratios are intimidating for many people in an exam setting. Yet, we use ratios in "real life" - inches to miles on a map, or the ratio of ingredients in recipes in cooking. A ratio shows the relationship between two numbers. In this case, it shows the relationship between those who have dietary restrictions and ordered meals to all those who ordered meals. One out of every twelve people who have ordered meals has special dietary needs, so we need to examine the relationship between the numbers one and twelve, and apply it to the 1800 people who have ordered meals.

Special dietary needs	as	1	as	what number
All who ordered meals	12			1800

One way to do this is to cross multiply. To cross multiply, we multiply the top of one number by the bottom of the other.

$$\frac{1}{12} = \frac{?}{1800} \qquad \frac{1}{12} \quad \frac{?}{1800}$$

12 x what number ? = 1 x 1800

12 x ? =1800

? = 1800 / 150 = 150

(You have to divide the 1800 by the 12, because you want to "isolate" the ? on one side, since that will give you the answer. Since the 12 and the ? were being multiplied by each other, the only way you could “free” the ? was to move the12 over to the other side of the equal sign by dividing the 1800 by the 12. If you get as far as 12 x ? = 1800 but still can’t remember whether you should multiply or divide, you can still get the correct answer, because the difference between multiplying add dividing is so large that common sense will tell you which is right. In this case, dividing gives you 150 people, and multiplying gives you 270,000 (obviously too big a number). So, in a ratio problem, as long as you set up the relationship between the numbers involved correctly (part is to whole as part is to whole), you should be able to solve it.)

Another way some people do ratios is by remembering that "the product of the means equals the product of the extremes". This means that when you

multiply the 'Inside' numbers in a ratio problem together, and then multiply the "outside" numbers together, they will always equal each other. In this case, we would set it up like this-

1 is to 12 as ? is to 1800

The “inside” numbers, 12 and ?, would be multiplied together, 12 x ?, and would equal then "outside" numbers that have been multiplied together, 1800 and 1. So

12 x ? = 1800 x 1
12 x ? = 1800
? - 1800⁄12 = 150

All of this may have seemed totally unnecessary, but it's important to keep these methods in mind for other questions, when using them may be very helpful. Again, there are many ways to solve math problems. If you use different methods than those in this booklet, and your results are consistently correct, there's no need to change what you're doing.

3. The answer is D. This problem is actually easier than it may look at first, and there are a number of ways to do it. One way would be to first determine how much the office is paying per typewriter. We're told that it costs $360 to service 18 typewriters for six months. So, for a six month period, it would cost, per typewriter, the total amount, $360, divided by the total number of typewriters, 18. $360 ÷ 18 = $ 20 per typewriter. So it costs $20 to service each typewriter for a six month period. We need to find out how much it would cost to service six typewriters for a three month period. We know the service cost of each typewriter is $20 for six months. For three months, it would be half that amount, or $10 per typewriter. Since we are considering six typewriters, the cost would be six typewriters at $10 each, for a total of 6 x $10 = $60.

   Or we could have said that we're being asked to find the cost of one third of the typewriters (6 is 1⁄3 of 18), for half the time (3 months is half of six months). One third of the typewriters would cost 1⁄3 as much: 1⁄3 x $360 = $120, and since the time involved is 1⁄2 of the total

time, 1⁄2 of $120 equals $60. or we could have set it up using fractions: 360 x 1⁄2 x 1⁄3 = 360⁄6 = 60.

4. The answer is A. Many people miss this question, because they aren't quite sure what to do with the 15% figure they're given. We are told that $480 is 15% of the office's non-personnel expenses for December. So in order to find the answer we need to know what number 480 is 15% of. So, we are asking "480 is 15% of what number?". To find this, we divide 480 by 15%.

480 ÷ 15% = 480 ÷ .15 = 3200

It's always a good idea to go back to the problem and check our answer to see if it makes sense. Is 480 15% of 3200? If we divide 480 by 3200, we get .15 = 15%. So 480 is 15% of 3200, and if we multiply 3200 by .15, we get 480.

Some people aren't sure whether to multiply or divide. Again, common sense should tell you which, in this case, if you had multiplied, you would have gotten $72 as an answer, which doesn't make much sense. It's important in these exams to step back and evaluate the reasonableness of your solutions , yet people often fail to do this.

It was also possible in this question to work backwards from the answers given to get the right answer. If you weren't sure how to do it, you could have multiplied each choice by 15%, to see which one was equal to $480. Choice A, 3200 x .15 = 480, would have become the obvious choice. This is a legitimate way to solve these types of problems Some people select Choice D, because they incorrectly multiply, and then aren't sure where to put the decimal points. If you get confused, we suggest you use the sales tax to help you remember. For example, a 7% sales tax reflects a tax of $.07 on every dollar. in the corner of your scrap paper you could write, 7% = .07; .07 = 7% or 8.25% = .0825; .0825 = 8.25%. (If you do this, when a percent like .0035% comes along on a word problem, you'll be able to convert it to decimal from more easily, especially if you're nervous. Consulting the sales tax example, we'd notice that 7% = .07 meant the decimal was moved two places to the left when going from percents to decimals, so .0035% would equal .000035.

Or, if we had to convert a decimal like .00046 to a percent, consulting the sale tax, we'd see that in the case of .07 = 7%, the decimal was moved two places to the right, so we'd do the same here. .00046 would then equal .046%. Sorry if this was unnecessary, but many people get confused when dealing with decimals and percents on an exam).

5. The answer is B. Many people put D as an answer, thinking that it's not possible to find the answer from the information given. The key thing to note here is that the question says Catherine bought an equal number of tickets. That means that the relationship between each of the ticket amounts will always be equal. There won't be more $11 tickets than $9.00 or $8.00 tickets. (lt's true that if the question didn't state there were equal amounts of tickets, D would be the correct answer). Since we know there are equal amounts of each ticket, however, we can find the answer by: 1) adding up the cost of the tickets and 2) dividing this figure into the total dollar amount she paid for them. $11 + $9 + $8 = $28. She spent a total of $196, so we can find out how many of each she bought by dividing 196 by 28. 196 ÷ 28 = 7. Some people aren't sure how to do this problem at first but by spending time looking at the problem and playing with all the possible choices, it becomes clearer.

6. The answer is B. The first thing we need to do is find the number of males in Agency Y. Agency Y employs 13,800 people, 42% of whom are male. So we would multiply 13,800 by 42% to find the number of males. 13,800 x 42% is 13,800 x .42 = 5796 males in Agency Y. Of these, 50% are age 30 or younger. So half of the 5,796 males will be older than 30. 5,796 ÷ 2 = 2898 males older than 30.

7. The answer is D. There are manyways you could solve this problem. This is another ratio problem (see question #2). We're told a machine can collage 126 books of 400 pages each in 14 days. We need to find how many books it could collate in 30 days. Since it collates at the same rate, the answer will be in the same proportion as 126 is to 14.
So, ? / 30 = 126 / 14 There are many ways to do this.

One is to first find the relationship between 14 and 126. 126 is nine times 14, so the answer will be 9 times 30, or 270. Or, you could set up

a ratio and cross-multiply (see #2).

$$\frac{?}{30} = \frac{126}{14}$$

$$14 \times ? = 126 \times 30$$
$$14 \times ? = 3780$$
$$? = 3780 \div 14$$
$$? = 270$$

Or. you could have used "the product of the means equals the product of the extremes." (See Question #2)

14: 126 as 30: ?

$$14 \times ? = l26 \times 30$$
$$14 \times ? = 3780$$
$$? = \frac{3780}{14}$$
$$? = 270$$

8. The answer is C. Many people miss this question, because it's more difficult than it looks. Many will simply subtract $360 from $840, and get $480, choice A. Yet, if we go back through the problem and re-read it, we'll see that choice A can't be the correct answer. We know the typewriter and dictation machine together cost $840. We're also told that the typewriter cost $360 more than the dictation machine. So, to find the cost of the typewriter, and to check choice A, we should be able to add $360 to choice A's $480 to find the cost of the typewriter. $360 + $480 = $840. So, according to choice A, the typewriter costs $840. The total cost of the typewriter and dictation machine then becomes $360 + $840 = $1200. But the question states the totla of both was $840, so we know something is wrong. We're looking for an answer that will total $840 for the two objects, and that will also show that the typewriter is $360 more. In choice A, the typewriter comes out to be $360 more, but the total is not $840. There are several ways you can do this problem. A very legitimate way is to "work backwards" from the possible answers. You can take each choice, add $360 to it, and see if they total $840. If you do this, it becomes apparent that choice C is the answer. Adding the typewriter, $600, and the dictation machine, $240, they total $840.

The $600 typewriter is $360 more than the $240 dictation machine, so it checks out. This is a perfectly acceptable way to solve the problem.

There are many other ways to solve it. One is to use a little algebra.

$x + (x + 360) = 840$
$2x + 360 = 840$
$2x = 840 - 360$
$2x = 480$
$x = 240$

You don't need to know algebra to solve it, though. Another way to solve a problem like this is to take the difference between the total of the two numbers and the difference between them, and then divide by two. This will always give you the smaller number.

840 - 360 = 480. 480 ÷ 2 = 240. This method will always work, as it's the same mathematical operation used in algebra, without the algebra. Working backwards from the answers given will also always work.

9. The answer is A. This question looks more difficult than it is, primarily because unnecessary information is thrown in. The diameter of the wood is not needed. We know that half the length of the wood is needed for the back of the chair. Because the rest of the problem is given in inches, the easiest approach would be to convert 3⁄4 of a yard into inches. 3⁄4 yard equals how many inches? 3/4 yards equal 27⁄36 or 27 inches. We need to find what half of 27 inches is to find how many inches will be used
for pegs. 27 ÷ 2 = 13 1⁄2 inches. Since 13 1⁄2 inches are being used for the back of the chair, 13 1⁄2 inches are left for the pegs. Since each peg is 3⁄4 of an inch, we should divide the total length available, 13 1⁄2 inches, by 3⁄4 of an inch to find how many
pegs can be made. 13 1⁄2 ÷ 3⁄4 = 27⁄2 ÷ 3⁄4 (to divide fractions, we invert the second fraction and then multiply) 27⁄2 x 4⁄3 = 108⁄6 = 18 pegs. Or, if you hate fractions, you could have converted to decimals and then divided.

10. The answer is D. There are several ways to do a problem like this. One way to do this is to remember the following method. First, invert the two numbers you're given, four hours and seven hours. So these numbers become 1/4 and 1/7. Then, add them together. (1/4 + 1/7 do not equal 1/11; we have to find a common denominator first). Twenty-eight is a number both four and seven will divide evenly into.

$$\frac{1}{4} = \frac{7}{28} \quad \text{and} \quad \frac{1}{7} = \frac{4}{28}$$

$$\frac{7}{28} + \frac{4}{28} = \frac{11}{28}$$

The last thing needed is to invert again to find the answer. 28/11 = 2.545.

If you remember this method, invert, add together, and invert again, you will always be ably to answer this type of question. The problem can also be solved algebraically. Let x be the time it takes. Robin can wallpaper a room in 4 hours, and Susan can wallpaper a room in 7 hours. In x hours, Robin does x/4 part Of the work, and Susan does x/7 part of the work. Together they do the complete job.

$$\frac{x}{4} + \frac{x}{7} = 1.$$

(Multiply the equation by 28 to get rid of the fractions)

$$7x + 4x = 28$$
$$11x = 28$$
$$x = 28/11 = 2.545$$

Sometimes it's also possible to estimate an answer in these types of questions. When that isn't possible, it's good to use the first method or the algebraic method, as they always work with this type of work problem.

11. The answer is B. Most people find these types of questions irritating, not only because they bring back bad memories, but also because of their basic irrationality - who ever jogs, (or wallpapers, or builds chairs) at exactly the same rate every time. Nevertheless, it's important to know how to do these. In this problem, we need to first find Mary's speed. We can do this by dividing the distance she traveled (3 miles) by her time (40 minutes). 3 ÷ 40 = .075 miles per minute. We know Mary runs for

40 minutes. Alice runs the same distance in 30 minutes, so Mary has 10 minutes of running time left when Alice finishes. To find how much distance Mary has left, multiply her speed, or rate by the time left.

.075 x 10 = .75, or 3/4 of a mile.

12. The answer is A. We first need to determine how much the organization spent on each group of tickets.

50 $8 tickets = 50 x 8 = $400
25 $10 tickets =25 x l0 =$250
25 $15 tickets =25 x15 =$375

Since all the $8 and $15 tickets were sold at 25% above cost, the money spent on these tickets came back and the organization made a profit of 25%. So the total spent on these two groups of tickets was $400 + $375 = $775.
We need to find the 25% profit on these.

25% of 775 = .25 x 775 = $193.75.

$193.75 is the profit from the $8 and $15 group of tickets. Of the 25 ten dollar tickets, three were unsold. Since each ticket was worth $10, and there were three unsold, $30 was spent by the organization in buying these tickets that did not come back. So the $30 will have to be subtracted from what ever profit is made. We should then find the profit on the 22 ten dollar tickets that did sell. 22 x 10 = $220. 25% profit on $220 = .25 x 220 $55.00. There was a $55 profit on the ten dollar tickets sold. But we need to subtract the $30 worth of unsold tickets from this. 55 - 30 = $25 profit on ten dollar tickets, Remember that the organization made $193.75 on the other ticket sales. Adding $25 to $193.75, the total profit was $218.75.

13. The answer is C. Another percent problem. There are many ways to do this. One way to do this is to ask, $640 is 80% of what number? 640/ .80 = 800. (If you're not sure whether to multiply or divide, you could still figure it out. If you multiply you should notice that the answer you get, $512, is less than $640, so it couldn't be correct). Another way to do it would have been to set up a ratio.

$$\frac{640}{} = \frac{}{80}$$

? 100 (80% is to 100%)

80 x ? = 640 x 100, 80 x ? = 64,000, ? = 64,000/80

? = 800. Or, you could work backwards and go through each choice, multiplying by 20% and then subtracting this amount, to see which choice would give you $640. Choice C is $800. $800 x .20 = 160. $800 - $160 = $640.

14. The answer is A. It's first necessary to determine the total amount spent on travel costs in the three days. The total mileage was 145 + 72 + 98 = 315. The salesperson traveled 315 miles, at 21¢ per mile. So the cost is 315 x 21 = $66.15. We need to find what percent the travel costs were of sales. To do this we divide $66.15 by $2300.

66.l5 ÷ 2300 = .0287 = 2.87%.
Choice A, 3% is the closest of the four possible choices.

15. The answer is D. This is a tricky question because many people assume they must set up a ratio, and find the number of students. But the question is asking for the percent of students, not number of students. One out of every 8 students takes Latin. The number 650 is irrelevant. All we need to find is what percent 1 is of 8. 1 ÷ 8 = .125 = 12.5%. 12.5% is closest to choice D, 13. (They don't need to put a percent sign next to each choice, as the question is asking for an answer in percents.) Once again, this question shows the importance of reading the problem carefully.

16. The answer is B. We first need to determine the total value of the in-kind contributions by adding $14,500 and $1,200. 14,500 + 1,200 = $15,700 . Then we need to find what percent of the total of $102,000 the in-kind total of $15,700 is. We can do this by dividing the in-kind contributions by the total budget.

15,700/102,000 = .1539 = 15.4%

17. The answer is C. We know from question #16 that the program has a budget of $102,000. We can find the answer by setting up a

relationship between the pounds of food distributed to the money budgeted.

$$\frac{\text{lbs. of food}}{\text{money budgeted}} = \frac{250{,}000}{102{,}000} = 2.45$$

There were 2.45 pounds of food distributed for each dollar spent.

18. The answer is A. One way to do this is to first find how many cases of jars were purchased. Jars are sold in cases of 12, not individually. If there are 76 quarts of tomatoes, dividing 76 by 12, we get 6.33 cases. Since we can t buy jars individually, we need 7 cases. For 20 pints of jelly, we'll do the same, dividing 20 by 12. 20 ÷12 = 1.66, so we'll need 2 cases. We'll need a total of 9 cases of jars. The regular price per case of jars is $4.25. But there is a discount of 15%.
15% of 4.25 = .15 x 4.25 = .637 = .64 (to the nearest cent).
$4.25 - the discount of .64 = $3.61 per case. There are 9 cases that are purchased at $3.61 per case. So the cost of jars alone is $3.61 x 9 = $32.49. Next we have to find the cost of the processing. The cannery charges 15¢ per quart. There are 76 quarts at 15¢ per quart, so the cost of processing equals 76 x .15 = $11.40. There are 20 pints that need to be processed at a cost of 25¢ for 2 pints, so the cost of processing will be 20 x .25⁄2 = $2.50. So, the cost of the canning will be the total of the cost of the jars, $32.49, the cost of processing 76 quarts , $11.40, and the cost of processing 20 pints , $2.50. $32.49 + 11.40 + 2.50 = $46.39.

19. The answer is A. This is a different kind of ratio problem. There are several ways to solve it. One way is to first add the parts given in the ratio in the problem. 1+ 2 + 7 =10. Then divide this number into the total amount of whatever substance you've been given. In this case, its 12 ounces of cough medicine. This will give the value of each part. 12 ÷ 10 = 1.2. Now, to find how many ounces of each ingredient is used, we would multiply 1.2, which represents one part, by the ratio of each of the ingredients given. Ingredient X is worth 1 part, so ingredient x = 1.2 x 1 = 1.2 oz. Ingredient Y is worth 2 parts, so 1.2 x 2 = 2.4 oz. Ingredient Z is worth 7 parts, so 1.2 x 7 = 8.4 oz . We can check to see if adding them would give us 12 ounces. 1.2 + 2.4 + 8.4 = 12.0 ounces, so it checks out. The question asks us for the amount of ingredient Y, 2.4 oz. (choice A). Another method is to express this algebraically: A + 2A + 7A = 12

$10A = 12 \qquad A = 12/10 \qquad A = 1.2 \qquad 2A = 2.4$

Or, you could work backwards from each choice, but in this case it's more work than using the above methods.

20. The answer is C. This is a percent increase question. We're told that the people served by Agency Y increased from 187,565 to 210,515. We need to find the percent increase. TO FIND PERCENT INCREASE OR DECREASE: 1.)Take the difference between the two numbers being considered, and 2.) Divide this difference by the original number, the number that chronologically came first. The difference between 210,515 and 187, 565 is 22, 950. 22,950 divided by 187, 565 (the earlier 1981, figure) equals .122 = 12.2%. If you can remember these two steps, you will always be able to answer this type of question.

21. The answer is B. There are several ways to do this. If, while doing these problems you use different methods, you shouldn't worry as long as your getting the right answers. There are many ways to approach these problems. One way to do this would be to set up an equation.

$$\frac{105 + 106 + 125 + ?}{4} = 116$$

The average of the three known numbers, and the unknown number equals 116. The above equation shows that if we add the four numbers together, and then divide by 4, we'll get 116. The 4 as a divisor is cumbersome. To get rid of it, we can multiply each side by 4.

$$\frac{4 \times (105 + 106 + 125 + ?)}{4} = 116 \times 4$$

$$105 + 106 + 125 + ? = 464$$

$$336 + ? = 464$$

$$? = 464 - 336$$

$$? = 128$$

You can check this if you wish by adding: 105 + 106 + 125 + 128 = 464. Dividing by 4, we get 116, so it checks out. Or, you could have solved this problem by working it backwards, taking each of the possible choices, adding it to the other three numbers, and then dividing by four to see if their average was 116. If you did this, which is a perfectly legitimate way to solve problems of this type, you would also have gotten 128, Choice B.

22. The answer is A. This problem can be solved without a lot of difficulty if the relationships between the workers and their times is kept clearly in mind. If it takes 16 typists 11 days to complete a project, we need to find how long it will take 10 typists working at the same rate. The 10 typists would complete the job 10⁄16 as quickly. So we could find the answer by dividing the days it took 16 typists, 11 days by 10⁄16.

11 ÷ 10⁄16 = 11 ÷ 5⁄8 11 x 8⁄5 = 88⁄5 = 17.6 days.

If you weren't at all sure how to do this, you may have wanted to first use a simpler example, so that you could then visualize what needed to be done. For example, what if the question had read “It takes 4 typists 8 days to complete a project. It would take 2 typists how many days?” You would have figured out that 2 typists would take twice as long, so it would have taken them 16 days. If you examined how you got this answer more carefully, you would be able to derive a method that could be used to solve the question. 2 typists is half of four typists. The 2 typists would complete the job 2⁄4 or half as quickly. The number of days it took was 8. You would then divide 8 by 1⁄2 to get the answer. 8 ÷1⁄2 = 8 x 2⁄1 = 16. If you weren't sure what you were supposed to do at this point, multiply, divide, or whatever, yet had a clue in that you knew the answer was 16. So you would do whatever would give you 16, and that was divide. This is a legitimate way to solve a problem, using a simpler, clearer relationship between two numbers, seeing how the problem would be solved, in that case and then applying the method to the test question. If you're stuck on how to approach a question, it's a good way to gain insight into how to solve it.

23. The answer is B. This is the same type of ratio problem as Question 19. The first thing we need to do is add the parts of the ratio together. 7 + 3 . 10. We then divide this into the total of the two numbers, 280. 280 ÷10 = 28. This means each part is equal to 28. The smaller number will equal 3 parts of 28, and the larger will equal 7 parts of 28.

3 x 28 = 84. 7 x 28 = 196

We're asked for the smaller number, 84. We can check this by adding 84 and 196 to see if they equal 280. 84 + 196 = 280.

24. The answer is D. Another percent problem. The 1982 population of Metropolis county is 130% of its l972 population, 145,000. So the population will be 130% x 145,000. (130% = 1.30). 1.30 x 145,000 =

188,500, Choice D. (If you weren't sure whether to multiply or divide, division would have given you a smaller number than the 1972 figure, Choice C, which wouldn't make sense since there was an increase, not a decrease, in population).

25. The answer is C. We know that the plane travels 10 miles a minute and the car travels 50 miles an hour. To find how far the car will travel when the plane travels 500 miles, we need to first find out how long it will take the plane to travel 500 miles. At 10 miles a minute, the plane will take 500 miles divided by 10 miles a minute,

$$\frac{500 \text{ miles}}{10 \text{ miles/ minute}} = 50 \text{ minutes.}$$

We need to find how far the car has traveled in 50 minutes. If the car travels 50 miles in 60 minutes, how far will it travel in 50 minutes? We can set up a ratio to find this.

$$\frac{50 \text{ miles}}{60 \text{ minutes}} \text{ as } \frac{? \text{ miles}}{50 \text{ minutes}}$$

One way to solve it is to notice that since this is a ratio problem, these numbers will be in direct proportion to each other. 50⁄60 is 5⁄6, So the answer will be 5⁄6 of 50.
5⁄6 x 50 = 41.66

26. The answer is D. We first need to find the total number of faculty. We know the ratio of students to faculty is 16:1, and there are 2,000 students. So we can find this by setting up a ratio.

$$\frac{\text{Students}}{\text{Faculty}} \text{ as } \frac{\text{Students}}{\text{Faculty}} \quad \frac{16}{1} = 2000$$

You can solve from here in a number of ways. One way is to observe that 2000 is 125 times greater than 16, so what we're trying to find will be 125 times greater than 1. Or, cross multiply:

16 x ? = 2000 x 1
16 x ? = 2000
? = 2000⁄16
? = 125

Or, use "the product of the means equals the product of the extremes", (see Question #2).

16 : 1 = 2000 : ?
16 x ? = 2000 x 1
? = 2000⁄16
? = 125

We know there are 125 faculty. If approximately 18% of them studied at the university then 82% did not. We need to find 82% of 125. 125 x .82 - l02.5 (Or you could have multiplied 125 by l8% and then subtracted the result from 125).

27. The answer is C. One of the few good things about multiple choice math questions is that the answer has to be one of the four given. In a problem like this, if you can't figure it out, it's possible to work backwards to get the answer. Most people select Choice A, because they misread and think $30 is the ten percent profit of the sales price of $300. But she bought the set for $300, and the question states that they sell all merchandise at cost plus 10%. If the profit was $30, the set would have to cost $300. But this would mean the total cost would be $330 . The cost, plus $30 (the 10% profit) added on, would total $330, not $300. So Choice A can't be correct. One way to do this would be to work backwards from each choice given. Choice B states that $27 is the profit. $27 is l0% of $270. Added together they equal $297, not $300, so Choice B is incorrect. Choice C is $27.27. $27.27 is 10% Of $272.70. $272.70 + 27.27 = $299.97. Since the answer says approximately, this looks like a safe choice. But just in case if you're not sure, we can check Choice D. $32.26 is 10% of $322.60, so we know that they won't add up to $300. Choice C is the answer. This is a perfectly good way to solve this problem. Arithmetic Reasoning is also testing your resourcefulness at working with numbers, and working backwards if you're stuck is certainly being resourceful. You could also say to yourself, $300 is 110% (100% is the cost of the item, plus a 10% profit added on) of what number? 300 ÷ 110% will give us the answer. 300 ÷ 1.10 = $272.72. So we know $272.72 is the actual cost. The profit will be 10% of this, or $27.27. Or you could have set up a ratio: 300⁄? = 110⁄100 (110% is to 100%) These methods are quicker than working backwards, so you may want to spend some time studying them.

28. The answer is B. We know that the temperature dropped 8° from midnight until 1 a.m., and before this it dropped at a constant rate. Working backyards from 1 a.m., we can add 8° to the temperature given at 1 a.m., 37°, to find the temperature at midnight. 37 + 8 = 45° at midnight. The temperature dropped at a constant rate from six p.m. to midnight. During that time it went from 54° to 45°. This is a drop of 9°. 54° - 45°, in 6 hours elapsed time from 6 p.m. to midnight. In 6 hours, the temperature dropped 9°. To find the rate the temperature dropped each hour, we would divide 9° by 6. 9° ÷ 6 = 1.5°. Since the temperature dropped 1.5° each hour, we can find the temperature for 10 p.m. by subtracting (4hours times 1.5 degrees), which equals 6° from 54. 54 - 6 = 48°, the temperature at 10 p.m. Or we could add two hours times 1.5 degrees, 3° on to the midnight temperature of 45°, 45 + 3 = 48°, Choice B.

29. The answer is A. We know that one eighth of a half gallon carton of ice cream has been eaten, and the remainder is divided by three people. The trick to this question, and it's a tricky question, is that they are asking for what percent of a gallon each, person gets, not of a half gallon, and there is only a half gallon of ice cream to begin with. So if 1⁄8 of a half gallon has already been eaten, we can find out how much this is by multiplying 1⁄8
by 1⁄2. 1⁄8 x 1⁄2 = 1⁄16. So 1⁄16 of a gallon has been eaten. The remainder is divided by three people. There was a half gallon, but 1⁄16 has been eaten. That leaves 1⁄2 - 1⁄16 • 8⁄16 - 1⁄16 = 7⁄16 left. The remainder, 7⁄16, is divided by 3 people.

7⁄16 ÷ 3 = 7⁄16 x 1⁄3 = 7⁄48

So each person gets 7⁄48 of a gallon. But the answer has to be expressed in percents. 7⁄48 as a percent is 7 divided by 48 =.1458, = 14.6%, Choice A. Fortunately, few of the exam questions are this tricky. It's always good to re-check your answers, with this type of question.

30. The answer is A. This question looks more difficult than it is. We know the woman got a score of "143 on a scale of 160". This means that out of 160 questions, she got 143 correct. We're asked to convert her score "to a scale of 0-100". All that means is that we're going to convert her score into a percent. (Percents are based on 100.) 143 is what percent of 160? To find this, we divide 143 by 160. 143 ÷ 160 = .893, or 89.4, Choice A.

31. The answer is C. This is another ratio problem. Since the assessment is at the same rate, we can set up a ratio between the numbers involved. (See Questions 2, 25, 26.)

$$\frac{\text{tax value}}{\text{tax value}} \text{ as } \frac{88}{28{,}000} \text{ as } \frac{110}{?}$$

or

$$\frac{\text{tax}}{\text{value}} \text{ as } \frac{\text{tax}}{\text{value}} \text{ as } \frac{88}{100} \text{ as } \frac{28{,}000}{?}$$

You can then solve it. If you cross multiply:

$$\frac{88}{110} = \frac{28{,}000}{?}$$

$$88 \times ? = 28{,}000 \times 110$$

$$88 \times ? = 3{,}080{,}000$$

$$? = 3{,}080{,}000 / 88$$

$$? = 35{,}000$$

32. The answer is B. Here's another work problem (see question 10). For the type of work problem that asks you to combine the efforts of two different people, there are several approaches you can use. One way to do this is to first invert the two numbers you're given, 14 days and 16 days. They become 1/14 and 1/16. Then, add them together. You'll need to find a common denominator to do this. 1/14 + 1/16 = 8/112 + 7/112 = 15/112. Then invert again to find the answer. 112/15 = 7.466 days.

33. The answer is D. This is another tricky question. It's important to remember that they got together to settle their accounts. This means they were paying each other back. It's a good idea to break this problem down into parts. A good way to do it is to write each person's name out, with the amount they are paying back or receiving directly under the name. So, Cynthia loaned $35 to Mary. Since they've gotten together to pay each other back, Cynthia will be getting back $35 from Mary. So we'll put +35 under Cynthia's name, and a -$35 under Mary's. Cynthia borrowed $14 from Jean: -14 for Cynthia (she's paying Jean back), +14 for Jean. She also borrowed $16 from Emily, so -16 for Cynthia, + 16 for Emily. Emily owes $17 to Jean: -17 for Emily, + 17 for Jean. Emily owes $9 to Mary: -9 for Emily, +9 for Mary.

Cynthia	Jean	Mary	Emily
+35	+14	-35	+16

	-14	+17	+9	-17
	-16			-9
Total	+5	+31	-26	-10

Emily left with $10 less than she came with. A lot of work for a point and a half.

34. The answer is D. We know the tiles are square, and that they are 12 inches on each side. To find out how many we'll , we need to find out how large the floor is. We're told it's 18 feet long and 11 feet wide. Multiplying 18 by 11 will give us the area that needs to be covered. 18 x 11 = 198 square feet. Since each tile is exactly a foot on each side, we'll need 198 of them.

35. The answer is C. Another percent problem. We know the car is presently valued at $3245, and that it's worth 72% of its original cost. To find this, we could say "72% of what number is $3245?"

72% x ? = 3245
.72 x ? = 3245
? = 3245/.72
? = 4506.94

Check it: is 3245 72% of 4506.94? 3245 ÷ 4506.94 = .72, or 72%. Or you could have set up a ratio.

3245/? = 72/100 (72% as to 100%) Or you could have worked backwards, taking 72% of each of the possible answers to see which would give you $3245.

36. The answer is A. Again, a percent problem. They're very common on this section. We know the sales tax rate is 4%, and the tax on the typewriter was $13.41. To find the purchase price before the tax was added, we could ask "13.41 is 4% of what number?"
13.41 is .04 of what number?
4%x? = 13.41
.04 x ? = 13.41
? = 13.41/.04
? = 335.25

To check it, multiply $335.25 by 4%. It should equal $13.41, and it does. Or you could have used a ratio.

13.41/? = 4/100 (4% is to 100%)

You could also have worked backwards, taking 4% of each answer until you got $13.41.

37. The answer is A. For an interest problem like this one, the interest will equal the rate x principal x time. The time is always expressed as some part of a year. In this problem, the interest will equal 8% x 600 x 1/12 (One month is 1/12 of a year).

.08 x 600 x 1/12 = 48 x 1/12 = 48/12 = $4.

The trick with these types of problems is to remember to express the time in terms of a year. In this case, the 30 days is expressed as one month, or 1/12 of a year.

38. The answer is B. We know the garden is 30 feet by 40 feet. We need to find the perimeter first, to determine how much fencing is needed. Remember, there are four sides. Two will be 40 feet, and two 30 feet. To find the perimeter, we need to add the four sides. 40 + 40 + 30 + 30 = 140 feet. It costs $1.75 per foot for the fencing. So the cost will be 140 x 1.75 = $245.

39. The answer is C. Many people miss this question. We need to find what part of her entire year's earnings she earns in April. We know she earns four times as much in April as in each of the other months. One way to do it would be to assign "parts". April would equal 4 parts. Each of the other months would equal one part, so eleven months with one part each would equal eleven parts. The total would then be 15 parts, 4 of which were April's earnings. So April's earnings would equal 4 out of the 15 total parts, or 4/15 Or, you could have assigned imaginary dollar values to see is the relationship more clearly. Imagine she made $1000 each month. April's earnings would be 4 times that, or $4000. In the other eleven months she'd make $11,000. The total would be $15,000. April's earnings would be $4000 of the $15,000, or 4000/15000. It sometimes helps to bring in "real life" examples to help see relationships more clearly.

40. The answer is D. Another ratio problem. A fitting ending. We're told a train travels 70 miles when a bus travels 50 miles. We need to find how many miles the train will travel when the bus travels 60 miles.

$$\frac{\text{train}}{\text{bus}} \text{ as } \frac{\text{train}}{\text{bus}} \qquad \frac{70}{50} \text{ as } \frac{?}{60}$$

One way to do this is to notice that 70 is 7⁄5 of 50, so the answer will be 7⁄5 of 60. 7⁄5 x 60 = 420⁄5 = 84

BASIC ALGEBRA

Algebra evolved from a simple and useful idea. When referring to an unknown number, it was more convenient to use a label or symbol to represent that number than to keep writing or saying "an unknown number." Mathematicians have given the unknown number a name, so that they can distinguish it from other numbers more easily. The most commonly used symbol or name for an unknown number, for our purposes, is x. But many letters of the alphabet (in Greek, Latin, italics, and lower and upper case) are used in algebra and statistics to represent or name both known and unknown numbers. Although it initially makes learning this material more difficult, once you've learned how to work with these combinations of letters and numbers, mathematical operations needed for word problems and statistics really do become easier to do.

In the first part of this booklet, we're going to be outlining all the rules and procedures you'll need to do basic algebra. It may all seem strange and irrelevant, but we'll be giving you the structural “building blocks”of algebra. Without these, it will be impossible to do the algebraic and statistical problems that appear on exams. We've also included a review of working with positive and negative numbers, since this knowledge is critical. We won't be getting to working with algebraic expressions, equations or statistics until much later in the booklet. Hopefully by that time you'll have a thorough understanding of the algebraic "basics", and you'll, be able to work with these much more easily. If some of this material seems familiar, please bear with us; we've designed this assuming no knowledge of algebra on the part of the reader.

SYMBOLS FOR THE FOUR FUNDAMENTAL OPERATIONS USED IN ALGEBRA

Except for multiplication, the symbols used in algebra are the same for thse used in regular math. It's very important to remember these.

Addition:	2+6 means 2 plus 6 x+y means x plus y
Subtraction:	4-1 means 4 minus 1 x-y means x minus y
Division:	6÷3 means 6 divided by 3 or 6/3 x÷y means x divided by y or x/y
Multiplication:	3•6 or (3) (6) or 3(6) or 3x6 means 3 times 6 x•y or (x)(y) or x(y) or xy means x times y 3•x or (3)(x) or 3(x) or 3x means 3 times x

ALGEBRAIC EXPRESSIONS

Since we're dealing with both letters and numbers in algebra, we have to be very careful to know exactly what operation we're performing when we use algebraic symbols.

For example, in using 3+x, we're adding 3 and x. If we use 3x, we're multiplying them (see above). We can also add x's: x+x+x=3x. For example, if x stands for the number 5, x+x+x would equal 5+5+5, or 15. 3x would equal 3 times 5, which is also 15. 3+x would equal 3+5, or 8. They could also be negative numbers: -x-x-x=-3x. Both 3x and -3x are examples of terms in algebra. A term is a quantity completely set off from other quantities to the left or right by either a plus or minus sign. In the example, 3x-4yz+8z, the 3x, 4yz, and 8z would all be considered terms (you have to imagine there's a plus sign in front of the 3x, since it's at the beginning of the expression and not a negative number). In the term 3x, the 3 and the x are called factors. Factors are numbers that are multiplied together. The 3x is one term and has two factors: (3) , and (x) . 4yz is one term and has three factors (4), (y), (z). c3z is one term and has three factors (c),(3), and (z). If you had a horrible looking thing like 5a(b-c) you would still have one term. There is a minus sign, but it's inside of a parentheses, which means the result of b-c will be multiplied by 5a, so that the term is still completely set off from other possible quantities (If you

are still confused, it will become clearer soon).

The factors that make up a term are called coefficients. In the term 3x, the number 3 is the coefficient of x, and x is the coefficient of 3. 3x and x3 mean the same thing. The numerical coefficient refers to the number in a term (not the letter or letters). In 4ab, 4 is the numerical coefficient. If there is no number in front of a term, then the numerical coefficient should always be understood to be one. y means ly, abc means labc, -x means -1x, -bc means -1bc.

Sometimes the factors which make up a term are the same. For example xxx would mean x is used three times, and would be multiplied by itself 3 times. If x equaled 5, then x would equal 5x5x5. x (which is really 5 in this case), was used as a factor 3 times. aaaa would mean a was used as a factor four times. Because this is rather cumbersome, algebra uses exponents. An exponent is written slightly above and to the right of the factor that is being repeated, and tells how many times the same factor is being repeated.

$x^3=xxx$, $y^4=yyyy$ $5^3=(5)(5)(5)$, $a^6=aaaaaa$.

Similarly, $4a^3y=4aaay$, $6a^3b^4=6aaabbbb$, $-3a^2b=-3aab$, $-12r^2s^3=-12rrsss$.

PLEASE REMEMBER THERE IS A DIFFERENCE BETWEEN EXPONENTS AND NUMERICAL COEFFICIENTS.

Numerical coefficients precede the unknown variables and act as a multiplier. Exponents cause the variable to be multiplied by itself one or more times such as:

$$4a = a+a+a+a = aaaa$$
$$3y = y+y+y = yyy$$

(a^3 would be pronounced “a cubed”, or “a to the third power”, x^2 would be pronounced “x squared”, or “x to the second power”, c^6 would be “c to the sixth power”, etc. Do not worry if you're not sure what all these a's, b's, c's, and y's represent. They merely represent unknown numbers. There's no way we could know what numbers they represent, because they're not in equations. We're just using them here to illustrate certain algebraic procedures.)

A REVIEW OF SIGNED NUMBERS

Before we continue on, it's best to briefly review positive and negative numbers, called signed numbers, and their operations. Positive numbers are numbers greater than zero. Negative numbers are numbers less than zero.

Addition of signed numbers

The sum of two positive numbers is always positive. Example: +3 plus +8 equals +11.

The sum of two negative numbers is always negative. Example: -7 plus -8 equals -15.

Adding numbers of unlike signs is a little more complicated, and some people have trouble with it. Try adding -24 and +16. If vou're not sure, you could think of money spent as opposed to money earned. If you spent $24, and earned $16, you would have a debt of $8, or -8. WHEN ADDING NUMBERS OF UNLIKE SIGNS, YOU SHOULD FIRST SUBTRACT THE TWO NUMBER. IF THE LARGER NUMBER IS POSITIVE, THE ANSWER WILL BE POSITIVE. IF THE LARGER NUMBER IS NEGATIVE, THE ANSWER WILL BE NEGATIVE.

If you're having a problem, try these for practice. (Answers are at end of section).

Add:

1)	(-6) + (2)	3)	(-467) + (421)
2)	(14) + (-16)	4)	(32) + (-13)

When adding more than two numbers, you would add up the like signs first, and then subtract. The sign of the answer will be determined by which sum of positive or negative numbers was larger. Example: (-189) + (52) + (-43) + (112). Adding -189, and -43, we get -232. Adding 52+112 equals 164. Subtracting 164 from 232, we get 68. Because the sum of the negative numbers was greater than the sum of the positive numbers, 68 will be expressed as a negative number, -68.

Subtraction of Signed Numbers

The rule for subtracting two signed numbers is straightforward: TO SUBTRACT TWO SIGNED NUMBERS, ADD THE OPPOSITE OF THE SECOND NUMBER. Another way to express it is to CHANGE THE SIGN OF THE NUMBER BEING SUBTRACTED AND THEN ADD.

Example: (-42) - (-33) would equal -42+33=-9. We changed the sign of the -33, because it was being subtracted, and then added the numbers together to get the answer. Some practice questions follow.

Subtract: (45)-(-23) You would first need to change the sign of the -23 to a +23 and then add the two numbers together. 45+23=68.

If you're having a problem, try these for practice. (Answers are at end of section).:

5) -32-(-28) 6) (16) - (-24)

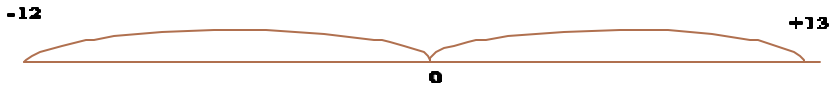
Example: At the beginning of the month, Alex's checking account had a total of $278.15. At the end of the month, the account was overdrawn by $43.75, excluding bank charges. If there were no deposits during the month, what was the total amount of the checks that Alex wrote?

To answer this, many of us would add the two amounts, $278.15 and $43.75, to get the answer, $321.90. What we could actually be doing here is subtracting the ending balance from the original balance. $278.15-(-43.75) would actually, following the method outlined above, turn into $278.15+ $43.75, as we would have changed the sign of the number being subtracted and then added the numbers together. One more example to illustrate this point.

Example: When you got up at 6 a.m. , the temperature was -12°F. At noon the temperature had risen to +13°F. What was the change in temperature from 6 a.m. to noon?

To find the change, we need to find the difference between the two temperatures, which means subtraction. We need to change the sign of the number being subtracted, and then add the numbers together, 13°-(-12°)=

25°. (If you weren't sure which to subtract from which, it wouldn't have



mattered, since all they were asking for was the change in temperature.)

A number line could also illustrate this.

First the temperature had to go up 12° to get from -12° to 0°, then increase an additional 13° to go, from 0° to 13°. Adding this distance, we get 25°.

We've included these examples to try and provide more of a context for these rules about working with positive and negative numbers.

MULTIPLICATION OF TWO SIGNED NUMBERS

When multiplying two numbers: IF THE NUMBERS HAVE THE SAME SIGN, THE PRODUCT IS POSITIVE.

Examples: (3)(4) = 12 (-4)(-6) = 24

IT'S IMPORTANT TO REMEMBER THAT THE PRODUCT OF TWO NEGATIVE NUMBERS IS POSITIVE.

IF THE NUMBERS HAVE DIFFERENT SIGNS, THE PRODUCT IS NEGATIVE.

Examples: (-3)(2) = -6 (17)(-2) = -34

If you're having a problem, try these for practice. (Answers are at end of section).:

7) (-8) (-18) 8) (14) (-7)

DIVISION OF TWO SIGNED NUMBERS

Division of two positive and two negative numbers is similar to multiplication.

THE QUOTIENT (RESULT) OF TWO NEGATIVE NUMBERS IS POSITIVE. THE QUOTIENT OF A POSITIVE AND A NEGATIVE NUMBER IS NEGATIVE.

Example: $(-8) \div (4) = -2$

If you're having a problem, try these for practice. (Answers are at end of section).:

Divide: 9) $(144) \div (-12)$ 10) $(-221) \div (-17)$

MULTIPLYING AND DIVIDING STRINGS OF NUMBERS

TO MULTIPLY OR DIVIDE STRINGS OF NUMBERS, ONE SHOULD FIRST MULTIPLY AND DIVIDE AS IF THE NUMBERS WERE UNSIGNED. Then, IF THERE IS AN EVEN NUMBER OF MINUS SIGNS, THE RESULT IS POSITIVE. IF THERE IS AN ODD NUMBER OF MINUS SIGNS, THE RESULT 1S NEGATIVE.

Example: (5) (-3) (-2) (-6) = -180 There are three minus signs, an uneven number, so the result is negative.

The Order of Algebraic Operations

In algebra, it's necessary to determine the order of operations when we have more than one operation in a problem. For instance, 40 ÷ 2 - 24 ÷ 3. We need to know when to do what, and the following rules should be followed. In an exercise with more than one operation, you should use the following rules:
(They apply for arithmetic as well).

1. Do what is inside the parentheses first. (Parentheses are also implied below and above any fraction bar).

2. Do exponents next.

3. Working from left to right do multiplications and divisions as you come to them.

4. Go back to the left and work to the right doing additions and subtractions.

So, it's: Parentheses
Exponents
Multiplication and Division
Addition and Subtraction

In the example, first perform division, then substraction. The problem becomes 20 - 8 = 12.

This is very important to remember. If you're having trouble you can memorize the order by thinking "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally." If you want something a little difference you could use "**P**lease **E**xcuse My **D**readful **A**lgebraic **S**kills," or "**P**lease **E**ncourage **M**y **D**aring **A**lgebraic **S**kills,"

depending upon your outlook.

Answers:

1) -4	6) 40
2) -2	7) 144
3) -46	8) -98
4) 19	9) -12
5) -4	10) 13

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