

VERTICAL CURVES

According to AASHTO "Vertical curves should be simple in application and should result in a design that is safe, comfortable in operation, pleasing in appearance, and adequate for drainage¹".

In order to describe this topic we need to understand some of the terms used.

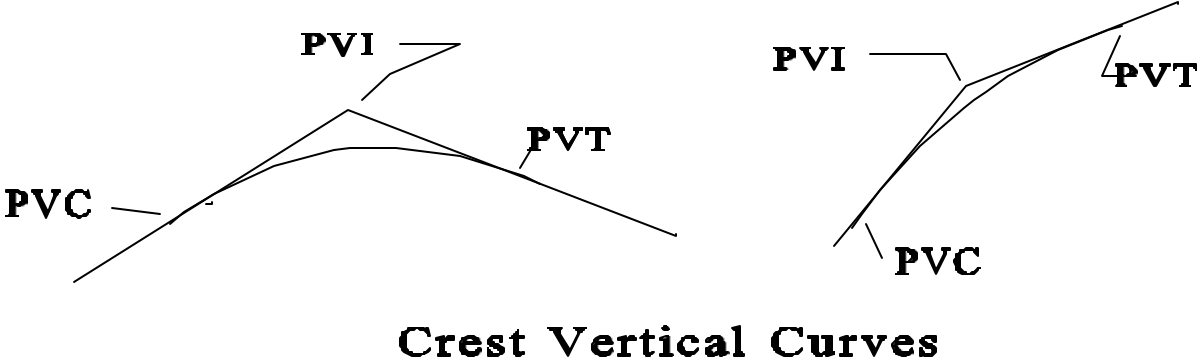
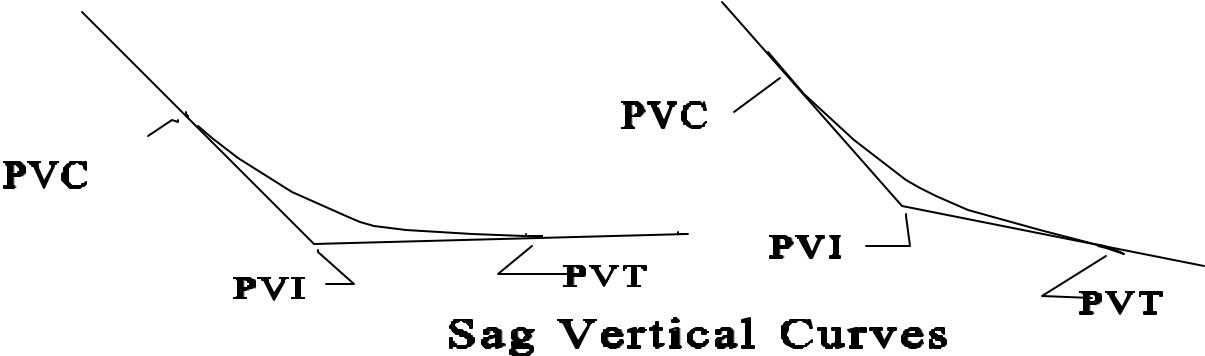
NYSDOT terminology	AASHTO terminology	Definition
PVI (Point of Vertical Intersection)	VPI (Vertical Point of Intersection)	Theoretical location where intersecting grades would intersect if there were not a vertical curve introduced. Generally described as Station and Elevation.
PVC (Point of Vertical Curve)	VPC (Vertical Point of Curvature)	Location where the Vertical Curve begins (as one proceeds in an up-station direction)
PVT (Point of Vertical Tangent)	VPT (Point of Vertical Tangency)	Location where the Vertical Curve ends (as one proceeds in an up-station direction)

We also need to know some of the general guidelines that are utilized, these include:

1. Vertical Curves used in highway design are Parabolic (not circular). There is a uniform rate of change of grade throughout the curve.
2. The Vertical Curves are almost always symmetrical (half the vertical curve length occurs on either side of the PVI)

¹AASHTO A policy on Geometric Design of Highways and Streets 1994

Vertical curves fall in one of two categories, sag or crest.



Sag Vertical Curves

The safety concern relative to Sag vertical curves is related to HSD (headlight sight distance). The following formulas are used to calculate HSD.

When S is less than L

$$L = (A S^2) / (120 + 3.5S)$$

When S is greater than L

$$L = 2S - (120 + 3.5S) / A$$

Where L = length of sag vertical curve, m

S = light beam distance, m; and

A = algebraic difference in grades, percent

Riding comfort is also a consideration in the design of Sag vertical curves. The general criterion is given as:

$$L = (A V^2) / 395$$

Where L & A are the same as above and:

V = the Design Speed

Crest Vertical Curves

The safety concern relative to crest vertical curves is related to SSD (Stopping Sight Distance) the distance at which a driver can see an object in the road ahead. The formulas for length of crest vertical curves for the standard eye height of 1070 mm and object height of 150 mm are as follows:

When S is less than L,

$$L = (A S^2) / 404$$

When S is greater than L,

$$L = 2S - (404 / A)$$

Where L = length of vertical curve , m;
 S = sight distance, m
 A = algebraic difference in grades, percent

Passing sight distance can also be considered on crest vertical curves. The same eye height as given above, coupled with an object height of 1300 mm, yields the following formulas for Passing Sight Distance.

When S is less than L ,

$$L = (A S^2) / 946$$

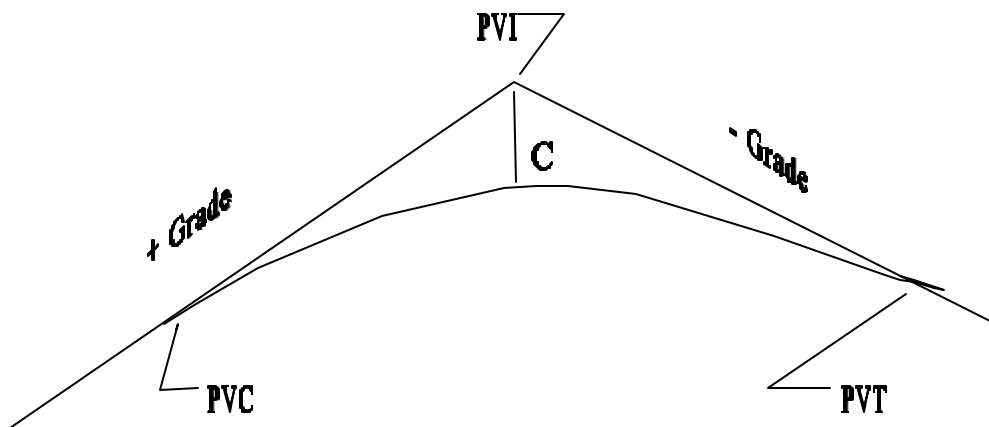
When S is greater than l ,

$$L = 2S - (946 / A)$$

AASHTO also provides minimum lengths for crest vertical curves based upon Design Speed and Algebraic difference of grades.

Points on a Vertical Curve

Elevations along a vertical curve can be computed using mathematical formulas.



$$C = \frac{\{(g_1 - g_2) / 8\} L}{100}$$

Where C = Center correction

g_1 = the grade back from the PVI

g_2 = the grade ahead of the PVI

L = the length of vertical curve (in meters)

note: $g_1 - g_2$ is the algebraic difference in grades

The correction at any point along the curve is defined by the formula

$$C_x = \left[\frac{(\text{Dist from PVT (or PVC whichever is closer) to sta. in question})}{\frac{1}{2} \text{the length of the curve}} \right]^2 \times C$$

HORIZONTAL CURVES (THEORY OF)

The horizontal circular curve is a part of a circle (an arc) connecting two straight lines which otherwise would intersect at an angle. Refer to the drawing on the next page concerning the following definitions.

The straight lines are called tangents. The point on a highway where the tangent stops and the curve begins is called the Point of Curvature or "P.C." abbreviated. The point where the prolongation of the two tangents intersect is called the Point of Intersection or "P.I.". The point where the curve stops and the tangent begins is known as the Point of Tangency or "P.T.". All these points proceed in the order of increasing stationing. That is: stationing of the P.C. is lower than that of the P.T.

The straight line from the P.C. to the P.I. is known as the Back Tangent, and the line from P.I. to P.T. is known as the Forward Tangent.

If perpendicular lines or radii are erected from the P.C. and P.T., they will intersect at the center of the circle forming a central angle usually designated as "I". This angle is equal to the angle formed by prolongation of the Back Tangent past the P.I. and the forward Tangent.

Angle BAC is known as the deflection angle at the P.C. not to be confused with the deflection angle at the P.I. This Deflection angle (at the P.C.) Is half of the Central angle.

The two tangents and two radii form two right triangles, whose legs are the radii and the tangents, and whose hypotenuse is the same identical line.

By the use of trigonometry and a few definitions it is possible to solve all circular curve problems without remembering too many formulae.

We can show this with an example:

Say $R = 500\text{m}$ and Deflection Angle $A = 20^\circ$

Find L = Length of curve

T = Tangent distance

E = External distance

M = Middle ordinate

L_c = Length of long chord

' L ' is most easily found using circular curve formulae

Using 360° in a circle and perimeter of a circle

is $2\pi R$: $L = \frac{I}{360^\circ} (2\pi R) = \frac{20}{360} (2\pi 500)$

$$L = \frac{2\pi (500)}{18} = 174.53\text{m}$$

Use trigonometry to find all other distances/length

' T ' = tangent length = $AB \tan \left(\frac{I}{2}\right) = \frac{T}{R}$

$$\text{or } T = R \tan \frac{I}{2}$$

$$T = 500 (\tan 10^\circ) = 88.1$$

' E ' = External distance = BD

Using Trigonometry Find OB

$$\cos \left(\frac{I}{2}\right) = \frac{R}{OB}$$

$$\text{Or } OB = \frac{R}{\cos \left(\frac{I}{2}\right)}$$

$$OB = \frac{500}{\cos 10^\circ} = 507.71 \text{ m}$$

$$\text{then } E = OB - R = 7.71 \text{ m}$$

M = Middle Ordinate = DH

Find OH using trig.

$$\cos \left(\frac{I}{2}\right) = \frac{OH}{R}$$

$$OH = 500 (\cos 10^\circ) = 492.40 \text{ m}$$

$$\text{Then } M = R - OH = 7.60 \text{ m}$$

L_c = Long Chord = AC

$$L_c = 2 (AH)$$

$$\sin \left(\frac{I}{2}\right) = \frac{AH}{R}$$

$$\begin{aligned} & R \\ AH &= R (\sin 10^\circ) = 86.824 \\ Lc &= 2 (86.82) = 173.65\text{m} \end{aligned}$$

LAYOUT OUT A CURVE

The most convenient method of locating points on a circular curve in the field is by means of deflection angles from the tangent.

Using the figure below:

Any circular curve can be divided into an infinite amount of segments. In all cases the deflection angle

from the tangent to the chord is 1/2 the central angle of the segment.

- i.e. $\angle BAC = \frac{1}{2} \angle ADC$ or $\frac{1}{2} I^\circ$
 $\angle BAG = \frac{1}{2} \angle ADG$ or $\frac{1}{2} M^\circ$
 $\angle BAH = \frac{1}{2} \angle ADH$ or $\frac{1}{2} P^\circ$
 \angle is symbol for angle

Therefore, any point along a circular curve can be found by turning a calculated deflection angle, based upon a predetermined segment curve length.

For example in the curve shown above say $R = 500$ m

$$I = 20^\circ$$

Previous example found $L = 174.53$ m

Find deflection angle to turn from PC using 15 m segment lengths - "X"

First find central angle for a 15 m chord length $\angle M^\circ = I^\circ \div L/X$ or $\frac{XI}{L}$

$$\angle M^\circ = \frac{(20^\circ)(15\text{m})}{174.53\text{m}} = 1.7189^\circ$$

Or $1^\circ - 43' - 8''$

$$\angle BAG = \text{deflection} \angle = \frac{1}{2} M^\circ = 0^\circ - 51' - 34''$$

$$\angle BAH = \text{deflection} \angle = \frac{1}{2} N^\circ = 1^\circ - 43' - 8'' \times 2 = 1^\circ - 43' - 8''$$

$$\angle BAJ = \text{deflection} \angle = \frac{1}{2} P^\circ = 2^\circ - 34' - 42'' \times 3 = 2^\circ - 34' - 42''$$

and so forth

This method can be used to find any point on the curve using any segment length.

Also, the deflection angle can be turned at any location on the curve if obstructions prohibit placing all the curve in from the PC.

For example say you couldn't see point J on the curve because of an obstruction in your line of sight. By setting up on pt H and sighting on the PC (pt A) turn 180° and then turn the originally computed deflection angle of $2^\circ - 34' - 42''$ (for the third segment) to find point 'J', 15 meters away from H. (See dotted lines).

If you couldn't see the PC (pt A) you could sight back on pt G, Turn 180° and then turn $(2^\circ - 34' - 42'') - (0^\circ - 51' - 34'') = 1^\circ - 43' - 8''$.

This logic can be used to find any point as long as you use the correct central angle.

HORIZONTAL CURVES

FORMULAS

Radius of 1 degree of curve = 5729.6 feet- Degree of curve just another way of stating radius.

$$(1) \text{ Radius of any Curve (R)} = \frac{5729.6 \text{ Feet}}{\text{Degree of Curve (D}_c\text{)}} \text{ (in feet)}$$

$$(2) \tan \frac{1}{2}A^\circ = \frac{\text{Tangent}}{\text{Radius}}$$

$$\text{Tangent (t)} = \tan \frac{1}{2}A^\circ (\text{Radius})$$

$$(3) \sin \frac{1}{2}A^\circ = \frac{\frac{1}{2}\text{Long Chord}}{\text{Radius}}$$

$$\text{Long Chord (Lc)} = 2 (\text{Radius}) (\sin \frac{1}{2}A^\circ)$$

$$(4) \cos \frac{1}{2}A^\circ = \frac{\text{Radius}}{(\text{Radius} + \text{External Distance})}$$

$$\text{External Distance (E)} = \frac{\text{Radius}}{\cos \frac{1}{2}A} - \text{Radius}$$

$$(5) \cos \frac{1}{2}A^\circ = \frac{(\text{Radius} - \text{Middle Ordinate})}{\text{Radius}}$$

$$\text{Middle Ordinate (M)} = \text{Radius} - (\text{Radius} \cos \frac{1}{2}A)$$

$$(6) \text{ Length of Curve (L)} = \frac{\text{Central Angle}^\circ}{\text{Degree of Curve (100')}} \text{ or } = (2\delta R) \frac{\text{Central Angle}^\circ}{360^\circ}$$

(7) Angle Deflections @ any station:

71 With transit set over P.C. Sight on the P.I.

72 Deflection/foot = $(\frac{1}{2} \text{Degree of Curve}) \frac{(\text{Distance from P.C.})}{100}$ or

$$\text{Deflection} < = \frac{\frac{1}{2} \text{ } ^\circ (\text{Distance from PC})}{L_c}$$

73 Deflection @ P.T. = $\frac{1}{2}$ Central Angle

HORIZONTAL SIGHT DISTANCE

Horizontal Sight Distance (SD) = $8 M R$ (Approx.)

Where M = distance from Centerline to obstruction and measured at eye height of driver.

Example: R = 500 m

and BLDG. LOCATION 10 m from C_L

$$SD = (8) (10) (500) = 40000 = 200 \text{ m}$$

HORIZONTAL/VERTICAL ALIGNMENT BASICS

2561 Introduction

10 Alignment related factors.

- | | |
|----------------------|--------------------------------------|
| 11 Safety | 5. Vehicle Operation/Maneuverability |
| 12 Aesthetics | 6. Riding Comfort |
| 13 Fuel Consumption | 7. Drainage |
| 14 Construction Cost | 8. Utilities |
| | 11 R.O.W. and Abutting Properties |

II Sight Distance - The length of roadway ahead visible to the driver.

- 11 Stopping Sight Distance - The distance a driver must see so that he is able to stop his vehicle to avoid striking an unexpected object on the traveled way.

Reference: AASHTO (page 117)

Criteria: Chapter 2 Highway Design Manual

01 Stopping Distance

- 11 Brake Reaction Time - Time it takes to apply the brakes after seeing a hazard (AASHTO says 2.5 seconds).

$$d = 0.278Vt$$

= distance traveled during reaction

- 02 Approximate Braking Distance - Distance to stop after applying brakes

$$d = \frac{V^2}{254}$$

($f \pm G$) where V = speed km/hr. f = coeff of friction between tires & roadway, g = grade

- 03 Example Problem (Attachment #1)

- 04 1994 AASHTO Tables III-1 & III-2 (Attachment #2)
- 11 Effect of Horizontal Curvature - Reference: 1994 AASHTO (Page 219)
 - 11 Application - Sight obstructions across the inside of curve.
 - 12 Example Problem (See Horizontal Curve Section)
- 03 Effect of Vertical Curvature - Reference: 1994 AASHTO (Page 283)
 - 11 Application - Sight obstructed by crest and headlight beam restricted by sag.
 - 12 Use NYSDOT Vertical Highway Alignment Sight Distance Charts
 - 1. Stopping sight distance for crest curve.
 - 2. Headlight sight distance for sag curve.
 - 13 AASHTO Figure III-3 - Graphical Solution (Attachment #3)
 - 14 See Vertical Curve Section for Sight Distance Formulas
- 02 Passing Sight Distance - Reference: 1994 AASHTO (Page 128)
 - 11 Application - Two lane - two way highway where faster vehicles desire to overtake slower vehicles.
 - 12 AASHTO Figure III - 2 (Attachment #4)
- III. Horizontal Alignment
 - 21 Horizontal Curves
 - Definitions and Formula (See Horizontal Curve Section)
 - 22 Control Of Vehicle
 - 11 Combination of horizontal curvature, superelevation and side friction to counterbalance centrifugal force
 - 11 From Laws of Mechanics: $\frac{e}{100} + f = \frac{V^2}{127R}$
 - e = superelevation (percent banking) v = velocity km/H
 - f = side friction factor r = radius (m)
 - 1994 AASHTO Table III - 6 (Attachment #5)
 - 12 Maximum Superelevation -
 - To prevent slipping in snow/ice conditions.
 - e max. = 4% - Urban Streets, arterials and at grade expressways.
 - e max. = 6% - All Rural facilities and urban freeways.
 - 13 Reverse Superelevation (Adverse Crown) - Permitted on very flat curves.
- 12 Transitions - Reference: Highway Design Manual (Chapter 5 and Tables 2-9, 2-10, 2-11.) 1994 AASHTO (Page 191)
 - 21 Runout and Runoff
 - 11 Tangent Runout - Go from normal crown to adverse crown removed or vice versa.

- 12 Superelevation Runoff - Go from adverse crown removed to full superelevation or vice versa..
- 13 AASHTO Figure III-16 (Attachment 6)
- 22 Spiral Transitions
 - 21 Spiral - Curve of constantly changing radius. Simulate vehicle path. See Horizontal Curve Section.
- 23 Flat Curves Without Spiral
 - 31 Use runoff lengths from tables.
 - 32 70% of runoff on tangent, 30% on curve.

IV. Vertical Alignment

11 Vertical Curves

1. Definitions (See Vertical Curve Section)

2. Formula

a. Basic Equation

$$\text{Parabola } Y = a x^2 + bx + c$$

b. Low point of a sag curve (high point of a crest)

$$X = g \frac{L}{A} \quad g = \text{Flatter Gradient} \quad L = \text{Length of Curve}$$

$x = \text{Distance From End} \quad A = \text{Algebraic difference between gradients}$

12 Grades - Reference: 1994 AASHTO (Page 227)

01 Maximum Grades - Reference: Criteria: Highway Design Manual Chapter 2

01 Trucks most affected: cars okay at 4 - 5%

02 Length of Grade - 15 km/h reduction in speed is critical length.

03 AASHTO Figure III-29 (Attachment #7)

02 Truck Climbing Lanes - Reference: 1994 AASHTO (Page 241) Design Manual (Page 5 - 27)

11 Criteria

1.) Upgrade Traffic Flow > 200 vph

2.) Upgrade Truck Flow > 20 vph

3.) One of following conditions:

a.) 15 km/h reduction in truck speed

- b.) Level of service E or F
- c.) Level of service reduction of 2 levels

03 Minimum Grades - Maintain for drainage.

11 0.5% is usual minimum in cut section.

12 In fill sections, level profile may be used if not curbed.

2565 Other Considerations

50 Combination of Horizontal and Vertical Alignment -

Horizontal and vertical alignments should not be designed independently. Their designs are complementary and are a study in compromise. Once the general location of a roadway is fixed, "the problem remaining is the specific design and harmonizing of the vertical and horizontal lines, such that the finished highway, road or street will be an economical, pleasant, and safe facility on which to travel."

The design speed of the facility determines limiting values for many elements such as curvature and sight distance and influences many other elements such as width, clearance, and maximum gradient.

Alignment coordination should begin with preliminary design during which adjustments can be readily made. This occurs in Phase I of the design process as alternates are chosen and developed at larger scales.

Some general considerations for proper horizontal alignment and profile are:

11 Curvature and grades should be in proper balance. Tangent alignment or flat curvature at the expense of steep or long grades and excessive curvature with flat grades are both poor design. A logical design that offers the most in safety, capacity, ease and uniformity of operation, and pleasing appearance within the practical limits of terrain and area traversed is a compromise between the two extremes.

12 vice versa, generally results in a more pleasing facility, but it should be analyzed for effect on traffic. Successive changes in profile not in combination with horizontal curvature may result in a series of humps visible to the driver for some distance, a hazardous condition as previously discussed. The use of horizontal and vertical alignments in combination, however, may also result in certain hazardous arrangements, as discussed later in this section.

13 Sharp horizontal curvature should not be introduced at or near the top of a pronounced

crest vertical curve. This condition is hazardous in that the driver cannot perceive the horizontal change in alignment, especially at night when the headlight beams go straight ahead into space. The hazard of this arrangement is avoided if the horizontal curvature leads the vertical curvature, i.e., the horizontal curve is made longer than the vertical curve. Suitable design can also be made by using design values well above the minimums for the design speed.

- 14 Somewhat allied to the above, sharp horizontal curvature should not be introduced at or near the low point of pronounced sag vertical curve. Because the road ahead is foreshortened, any but flat horizontal curvature assumes an undesirably distorted appearance. Further, vehicular speeds, particularly of trucks, often are high at the bottom of grades, and erratic operation may result, especially at night.
- 15 On two-lane roads and streets the need for safe passing sections at frequent intervals and for an appreciable percentage of the length of the roadway often supersedes the general desirability for combination of horizontal and vertical alignment. In these cases it is necessary to work toward long tangent sections to secure sufficient passing sight distance in design.
- 16 Horizontal curvature and profile should be made as flat as feasible at intersections where sight distance along both roads or streets is important and vehicles may have to slow or stop.
- 17 On divided highways and streets, variation in width of median and the use of separate profiles and horizontal alignments should be considered to derive design and operational advantages of one-way roadways. Where traffic justifies provision of four lanes, a superior design without additional cost generally results from the concept and logical design basis of one-way roadways.
- 18 In residential areas the alignment should be designed to minimize nuisance factors to the neighborhood. Generally, a depressed facility makes a highway less visible and less noisy to adjacent residents. Minor horizontal adjustments can sometimes be made to increase the buffer zone between the highway and clusters of homes.
- 19 The alignment should be designed to enhance attractive scenic views of the natural and manmade environment, such as rivers, rock formations, parks, outstanding buildings, and golf courses. The highway should head into rather than away from those views that are outstanding, it should fall toward those features of interest at a low elevation, and it should rise toward those features best seen from below or in silhouette against the sky.

51 Poor and Good Practice Illustrations - Reference: AASHTO (Page 298)

11 See 1994 AASHTO Figure III-43 (Attachments #8 to #11)

52 Compound Curves - Reference: 1994 AASHTO (Page 199), Maximum Successive Radii:

Mainline 1:15, Ramps 1:2

11 See Attachment #12

53 Broken Back Alignments (See Attachment #13)

- E. Stopping sight distance controls, several elements of geometry, especially vertical curve alignment as well as the proximity to roadways that lateral sight obstruction (such as fences, walls, barriers) can occur.

Stopping Sight Distance - the distance required for a driver to come to an emergency stop.

Driver's height of eye is 1070 mm, the height of object is 150 mm

$$S = .278Vt + \frac{V^2}{30(f \pm g)} \quad \text{Where } V \text{ is design speed (MPH)}$$

t is reaction time (2.5 sec.)
f is grade of roadway

Ex. For V = 100 km/H on
a 0.5% downgrade

$$S = .278 \times 100 \times \frac{2.5}{.254} + \frac{100^2}{2 \times .05} = 230 \text{ m}$$

Note: Table below does not adequately treat S on steep grades.

Attachment #1

Note: Table below does not adequately treat stopping sight distance on steep grades.

Attachment #2

Attachment #3

Attachment #4

d = Distance traveled during perception - reaction time and initial acceleration until encroachment of opposing lane.

d_2 = Distance traveled while passing.

d_3 = Clearance length to opposing vehicle.

d_4 = Distance traveled by opposing vehicle.

Attachment #5

Attachment #6

Attachment #7

Examples: Upgrade = 4%

Critical length = 1000'
(i.e. 10 MPH Reduction)

Upgrade = 5.5%
Length of Grade = 1000'

Then, Speed reduction = 15 MPH

02 Poor 7 Good Practice Illustrations:

Attachment #8

Attachment #9

Attachment #10

Attachment #11

03 Compound Curves

Attachment #12

Compound Curves are advantageous in affecting desirable shapes of turning roadways at at-grade

intersections and of ramps at interchanges. We must gradually “step-down” the circular gradually in order to avoid a “kinked” or forced alignment. In general, the design manual and AASHTO recommend a ratio between successive curve radii of a maximum of 1:1.5 for the mainline and 1:2 for ramps.

Attachment #13

The “broken-back” arrangement of curves (having a short tangent between curves in the same direction) should be avoided. Most drivers do not expect succeeding curves to be in the same direction. Use of spiral transitions or a compound curve alignment, is preferable for such conditions.